Simulation and Modeling of Nonlinear Magnetics

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A procedure for modeling and simulation of arbitrary nonlinear magnetics components is presented. A set of electromagnetic primitives is described as implemented in Verilog-A. The primitives include cores, gaps, and windings that are combined to model ferromagnetic inductors and transformers. The physics of the ferromagnetic core model is described. The model is used to illustrate how to overcome some difficult modelling issues such as hysteresis, incremental models, implicit models, and multidisciplinary models.

Originally written in August 1994, this paper was presented at the IEEE International Symposium on Circuits and Systems (ISCAS-1995) in May 1995. In March 2002 the models were converted to Verilog-A.

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1 Introduction

Many designs for power applications, such as switching power supply and regulator circuits, make use of ferromagnetic inductors and transformers. These applications have created a need for a model that can accurately describe the operation of nonlinear magnetic devices. Unfortunately, these devices exhibit several qualities that make modeling difficult. In particular, the nonlinear hysteric nature and the wide variety of core topologies pose a challenge to modeling of such devices.

It is impractical to model all possible configurations with a single model topology. Instead, a transformer model of arbitrary complexity can assembled using a small set of building blocks composed of windings, cores, and gaps.

This paper describes a methodology to represent nonlinear magnetics devices. Models are developed to describe the operation of each of the building blocks. Linear relationships are used to describe the winding and gap models. The nonlinear model for the core is based on that of Jiles and Atherton [1][2]. Models for the electromagnetic primitives are implemented in Verilog-A. Finally, the results from a simulation of a transformer is presented.

2 Modeling Approach

Various transformer configurations can be modeled by treating the magnetic portion of the component as a circuit itself. The circuit consists of cores, gaps and windings interconnected in a topology that matches that of the component being modeled. The magnetic components relate magnetic force ($\mathcal{F}$) and flux (Φ). The equations for the composite component are assembled from the defining relations for each primitive component and from the observation that the flow of magnetic flux out of a node must be zero and the sum of the magnetic force around a loop also must be zero. Cores and gaps are similar in that they are purely magnetic components, whereas the windings represent a coupling between the electrical and magnetic circuits.

Consider a transformer with four windings constructed with an **E** core with a gap in the center arm. Physically, the transformer is shown in Figure 1. Schematically, the same transformer is shown in Figure 2. C1, C2, and C3 represent core fragments that model the three arms of the **E** core. G1 represents the gap in the center arm and G2 represents the leakage through the air around the windings. W1, W2, W3, and W4 are the four windings

3 Linear Magnetics Primitives

In order to understand the theory behind these components, it is helpful to first review the relationships between the fundamental electric and magnetic quantities. Figure 3 depicts the relationships between each of the electrical quantities and the analogous relationships between their magnetic counterparts. The transformations between these two domains are defined by Faraday's Law and Ampere's Law.

FIGURE 1 *Physical representation of an E core.*

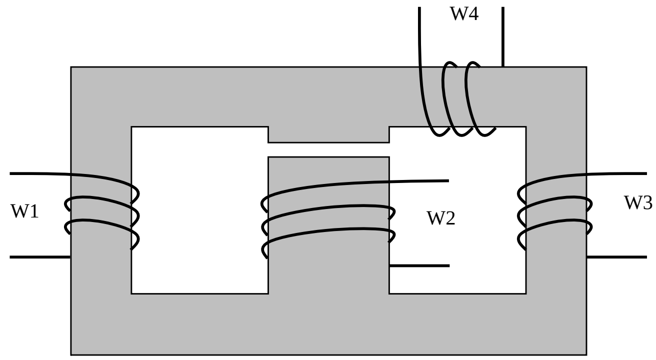


FIGURE 2 *Schematic representation of an E core.*

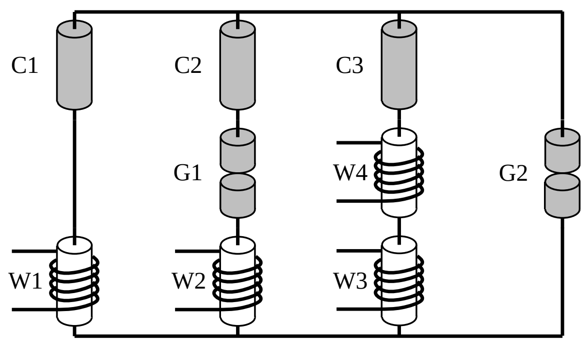
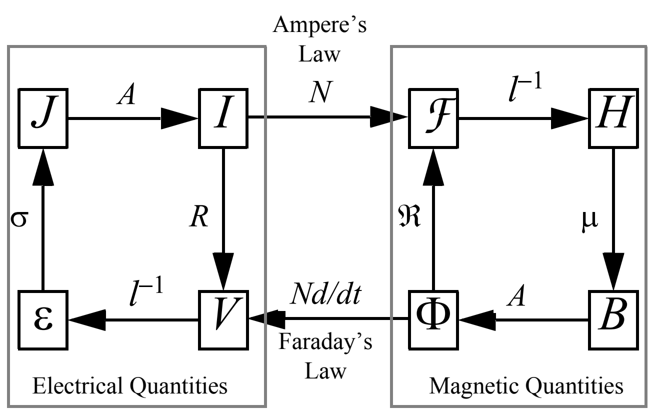


FIGURE 3 *Relationships between electronic and magnetic quantities.*



3.1 The Gap Model

An air gap in a magnetic circuit is analogous to a linear resistor in an electrical circuit. It is represented by a linear reluctance, $\Re$, that is a function of the cross-sectional area of the core, the gap length, and the permeability of free space, μ_0. From the figure, it is seen that the reluctance is the relationship between the magneto-motive force, $\mathcal{F}$, and the flux, Φ. The mathematical relationship is shown in Equation 1.

$$\Re_G = \frac{\mathcal{F}}{\Phi} = \frac{l}{\mu_0 A} \tag{1}$$

3.2 The Winding Model

A winding couples the electrical and magnetic domains. The energy transferred between domains is scaled by the number of turns in the winding, *N*. The relationship from the electrical to the magnetic domain is defined by Ampere's Law as shown in Equation 2.

$$\mathcal{F} = NI \tag{2}$$

Similarly, the energy transfer from the magnetic back to the electrical domain is defined by Faraday's Law. In addition, the electrical winding can have a finite series resistance, R_W, defined by Ohm's Law. These effects are combined via superposition, resulting in the relationship shown in Equation 3.

$$V = R_W I + N\frac{d\Phi}{dt} \tag{3}$$

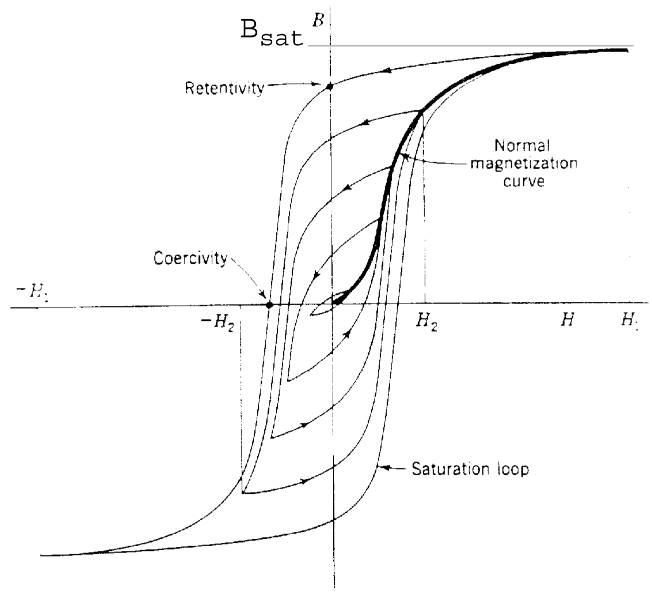
4 Nonlinear Magnetics Theory

This section describes the underlying theory of the nonlinear behavior of ferromagnetic devices. It is a phenomenological model based on the work of Jiles and Atherton [2]. While the magnetic quantities analogous to voltage (*V*) and current (*I*) are magneto-motive force ($\mathcal{F}$) and magnetic flux (Φ), the behavior of a saturable magnetic core is typically described in terms of its magnetic flux density (*B*) vs. magnetic field intensity (*H*). This relationship takes the form of a bistable sigmoid, as shown in Figure 4.

Several figures of merit for a device can be taken from its *B-H* curve, the most important of which are the saturation level, the remanence (or retentivity), and the coercivity (or coercive force). The saturation level (B_{sat}) is the magnetization level at which all of the domains' magnetic moments are pointing in the direction of the external field vector. This level, a constant for the material, represents the maximum field density the material can sustain. The remanence is defined as the amount of magnetism that remains in the material after the external forces have been removed. On the *B-H* curve, it is the point at which the curve crosses the abscissa (*H* equal to zero). The coercive force is defined as the external magnetic field required to return the magnetism of the solid to zero. On the *B-H* curve, it is the point at which the curve crosses the ordinate (*B* equal to zero). These parameters relate physical occurrences within a core to its magnetic response and provide limits by which mathematical relationships can be derived to model the magnetic behavior within the core.

The relationship between *B* and *H* is defined by the permeability (μ) of the material, as shown in Figure 3. For magnetic materials, the permeability (μ) can be related to the permeability of free space (μ_0) by the relative permeability (μ_r). A particular class of magnetic materials (saturable) can exhibit highly nonlinear relative permeabilities. The relationships between these quantities is shown in Equation 4.

FIGURE 4 *B versus H curve with minor loops.*



$$B = \mu H = \mu_0 \mu_r H = \mu_0 (H + M) \quad (4)$$

The parameter M in the equation refers to the magnetism or field intensity within the material that is contributed by the magnetic domains. The variation in permeability (μ) is a result of the changing magnetism (M) of the solid as energy from the applied field (H) is absorbed into the domains. Since the relative permeability (μ_r) is dependent on the ratio of applied and resident fields (magnetic susceptibility), the field density equation may be expanded into the more general relationship given by the final relationship, in which the relative permeability is incorporated into the internal field intensity component (M).

If a sample of magnetic material is examined on a per domain basis, the differential field strength around any given domain will be somewhat larger than expected due to its proximity to the remainder of the domains in the material. In effect, a given domain experiences the magnetic influence of the averaged total magnetism of the solid, since the orientation of any given domain may be random. At this point, it is appropriate to define an effective magnetic field intensity (H_{eff}), existing within the solid, that is the sum of the applied field (H) and some averaged contribution from the magnetism (M) of the surrounding domains. The proposed equation adjusts the percentage of bulk magnetism (M) added to the applied field intensity (H) through the scaling coefficient alpha (α) which typically has a value around 10^{-3}. The modified relationship for the magnetic field intensity experienced by a single domain is given by Equation 5.

$$H_{eff} = H + \alpha M \quad (5)$$

If a magnetic material was able to return all of the magnetic energy that was input, the resulting magnetization curve would take the form of a single valued sigmoid (equivalent to the center line of the hysteresis loop shown in Figure 4). This curve, referred to as the anhysteric magnetization curve, represents the ideal or lossless magnetization of a material. The function that was chosen to model this semi-empirical representation was developed by Langevin [5]. The parameters used to calculate this quantity are the effective field strength (H_{eff}) given by Equation 5, the saturation level (M_{sat}), and the shaping coefficient gamma (γ), which adjusts the slope of the curve according to the magnetic hardness of the material. The phenomenological representation of anhysteric magnetization (M_{anh}) proposed by Langevin is defined by Equation 6.

$$M_{anh} = M_{sat}\left(\coth\left(\frac{H_{eff}}{\gamma}\right) - \frac{\gamma}{H_{eff}}\right) \quad (6)$$

The total magnetic field in ferromagnetic materials can be separated into ideal and irreversible components, where the irreversible represents the energy dissipated in magnetizing the material. Jiles and Atherton formulated a model which expresses the total magnetization as a weighted average in the form shown in Equation 7.

$$M = cM_{anh} + (1-c)M_{irr} \quad (7)$$

The equation is composed of an anhysteric term which is a true function, in the mathematical sense, of the applied magnetic field (H) defined implicitly through the Langevin function, and an irreversible term which is a contribution computed through inexact differentials. The fundamental differential equation, proposed by Jiles and Atherton, is shown in Equation 8. The underlying derivation is beyond the scope of this paper, but can be found in the references [1][2].

$$M = M_{anh} - \delta k \frac{dM_{irr}}{dH_{eff}} \quad (8)$$

The equation consists of an energy term for the anhysteric component (representing the lossless term) and an energy term for the irreversible component that lumps the material dependent variation into a single term (k). The hysteresis is implemented using $\delta = \mathrm{sgn}(dH/dt)$, which takes on the values of either 1 or −1 depending on whether it is the positive or negative half cycle of the hysteresis curve. This equation can be converted to the standard differential notation by substituting Equation 5 into Equation 8 for the effective field (H_{eff}) and rearranging terms. The resulting form is shown in Equation 9.

$$\frac{dM_{irr}}{dH} = \frac{M_{anh} - M_{irr}}{(\delta k - \alpha(M_{anh} - M_{irr}))} \quad (9)$$

Finally, the total differential susceptibility can be obtained by combining Equation 7 and Equation 9. The result is shown in Equation 10.

$$\frac{dM}{dH} = (1-c)\left(\frac{M_{anh} - M_{irr}}{(\delta k - \alpha(M_{anh} - M_{irr}))} + c\frac{dM_{anh}}{dH}\right) \quad (10)$$

It became evident, after some work simulating various core profiles, that a condition can arise that yields non-physical results. This condition is exemplified by an unexpected increase in total magnetization (M) just after the input current changes direction (near the loop tips). After further investigation, including several conversations with Dr. David Jiles, it was found that the cause was the inappropriate calculation of the total magnetization between the point at which the current changed direction and the point where the magnetization curve crossed the anhysteric curve. It was decided that the magnetization should be completely reversible during this portion of curve. This is justified since the domain walls, once deformed in a particular direction by the applied field, must first relax to equilibrium and then be deformed in the opposing direction before any further irreversible change occurs. In order to accommodate this change to the equations, it is necessary to include an additional conditional clause to obtain the correct calculation of the total magnetization.

5 Verilog-A Model

Three Verilog-A [3,4] modules were developed based on the theory derived in the previous sections. The gap, winding and core module definitions are listed in Listings 1, 2, and 3, respectively.

The Verilog-A language provides pre-defined types for electrical and magnetic signals. A *discipline* describes the two types of signals associated with a node or a branch. Each signal types is referred to as a *nature*. Each discipline contains two natures, one for the potential signal, and one for the flow signal. The electrical discipline includes voltage (*V*) as the potential and current (*I*) as the flow. For the magnetic discipline the potential is magneto-motive force (*MMF*) and the flow is magnetic flux (*Phi*). Given a node or a branch (defined by a pair of nodes), functions with these names are used to access their respective signals (for example, *V*(*a,b*) is the electrical potential and *I*(*a,b*) is the electrical flow between nodes *a* and *b*).

The air gap module in Listing 1 is a direct implementation of the relationship shown in Equation 1. Note that the simplicity of this language allows for very concise specification of mathematical relationships. The ability to pass in arguments (*len* and *area*), with optional default values and bounds checking, allows for generic use of the module.

LISTING 1 *Air gap model.*

```
module gap(p,n);

magnetic p, n;
parameter real len=0.1 from [0:inf);              // effective length
parameter real area=1 from (0:inf);               // area

    analog begin
        MMF(p,n) <+ len * Phi(p,n) / (`P_U0 * area);
    end
endmodule
```

The winding module in Listing 2 directly implements the relationships defined by Equation 2 and Equation 3 which couple the magnetic and electrical domains. Note that the *ddt* operator is used to perform the time derivative.

The *core* module in Listing 3 implements the relationship between $\mathcal{F}$ and Φ for the nonlinear hysteretic core. Equation 10 describes the characteristics of the magnetic material in terms of incremental quantities, and so cannot be used directly in Verilog-A. However, the equation can be converted into something that can be used by first multiplying both sides by *dH* and integrating.

LISTING 2 *Electromagnetic winding model.*

```
module winding(ep,en,mp,mn);

electrical ep, en;
magnetic mp, mn;
parameter real turns=1;
parameter real r=0;          // winding resistance per turn

    analog begin
        MMF(mp,mn) <+ turns * I(ep,en);
        V(ep,e2) <+ turns * (r * I(ep,en) – ddt(Phi(mp,mn)));
    end
endmodule
```

$$\int dM = \int \frac{1}{1+c} \frac{M_{anh} - M_{irr}}{\delta k - \alpha M_{anh} - M_{irr}} dH + \int \frac{c}{1+c} dM_{anh} \qquad (11)$$

Finally, replace *dH* with *dH/dt dt* and simplify.

$$M = \int \left(\frac{1}{1+c} \frac{M_{anh} - M_{irr}}{\delta k - \alpha M_{anh} - M_{irr}} \frac{dH}{dt} \right) dt + \frac{c}{1+c} M_{anh} \qquad (12)$$

Notice that this equation is implicit because M_{irr} appears on both sides of the equal sign. This presents no problem because Verilog-A allows implicit formulations.

There are a few aspects of this implementation that should be explained. First, $\mathcal{F}$ = MMF(p,n) and ϕ = Phi(p,n). Second, to avoid division by zero in Equation 6 when H_{eff} is near zero, it is replaced by the first term in its Taylor series expansion when $|H_{eff}| < \alpha/(1000)$. The ^ operator is the logical exclusive or operator, so migrating is true if $\delta > 0$ and $M < M_{anh}$ or if $\delta < 0$ and $M > M_{anh}$. Finally, idt computes the time-integral of its first argument, and the second argument specifies the initial value of the integral.

6 Results

The circuit in Listing 4 was simulated using Spectre®[1]. The resulting waveforms depicting the voltage and current of the equivalent inductor and the resulting magnetization loop versus magnetic field intensity are shown in Figures 5 and 6, respectively. The electrical characteristics clearly show the collapse of the winding voltage and the rapid increase of current as the core saturates. The magnetic characteristics shown define the initialization, starting from an unmagnetized state, followed by its saturated hysteresis loop. This simulation required approximately 200 time points to generate the two cycles of response shown.

7 Conclusions

In this paper, we described a methodology of modeling arbitrarily complex electromagnetic components by combining primitive magnetic and electromagnetic components.

1. Spectre is a registered trademark of Cadence Design Systems.

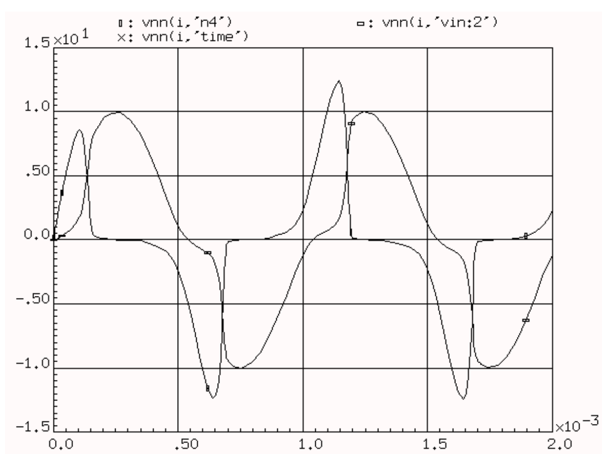
LISTING 3 *Nonlinear magnetic core model.*

```
module core(p,n);

magnetic p, n;
parameter real len=0.1 from (0:1000);        // effective magnetic length of core
parameter real area=1 from (0:inf);          // magnetic cross-sectional area of core
parameter real ms=1.6M from (0:inf);         // saturation magnetization
parameter real a=1100 from (0:inf);
parameter real k=2000 from (0:inf);          // bulk coupling coefficient
parameter real alpha=1.6m from (0:inf);      // interdomain coupling coef.
parameter real c=0.2 from [0:1];             // coef. for reversible magnetization
magnetic Hdot;          // internal Hdot node
real H;                 // field intensity
real B;                 // flux density
real Manh;              // anhysteric magnetization
real Mirr;              // irreversible magnetization
real dMirr;             // dMirr/dH
real M;                 // total magnetization
real Heff;              // effective field intensity
integer delta;          // direction of the input MMF
integer migrating;      // flag indicating that the pinning sites are moving

    analog function integer sign;          // sign function
        real arg; input arg;
        sign = (arg >= 0.0 ? 1 : –1);
    endfunction

    analog function real coth;             // hyperbolic cotangent function
        real arg; input arg;
        real x;
        begin
            x = exp(min(80,max(–80,arg)));
            coth = (x+1/x)/(x–1/x);
        end
    endfunction

    analog begin                           // core model
        H = MMF(p,n) / len;
        MMF(Hdot) <+ ddt(H);
        delta = sign(MMF(Hdot));
        B = Phi(p,n)/area;
        M = B/`P_U0 – H;
        Heff = H + alpha ∗ M;
        if (abs(Heff) > 0.001 ∗ a)
            Manh = ms ∗ (coth(Heff/a) – a/Heff);
        else
            Manh = ms ∗ Heff/(3.0∗a);              // Taylor series expansion of above
        dMirr = (Manh – M)/(delta∗k – alpha∗(Manh - M)) ∗ MMF(Hdot);
        migrating = (delta > 0) ^ (M > Manh);
        Mirr = idt( migrating ∗ dMirr, Manh );
        M = (1–c)∗Mirr + c∗Manh;
        Phi(p,n) <+ area∗`P_U0∗(H+M);
    end
endmodule
```

LISTING 4 *Spectre netlist of test circuit.*

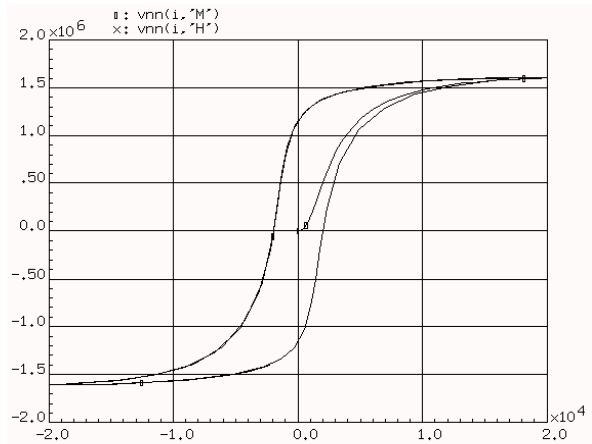
```
// Test circuit for Jiles and Atherton magnetics model implementation

simulator lang=spectre
ahdl_include "jilath.va"

// Magnetic Circuit
C1 (m m1) core len=60mm area=4u ms=1.7M a=1100 k=2000 alpha=1.6m c=0.2
C2 (m m2a) core len=20mm area=8u ms=1.7M a=1100 k=2000 alpha=1.6m c=0.2
C3 (m m3a) core len=60mm area=4u ms=1.7M a=1100 k=2000 alpha=1.6m c=0.2

G2 (m2a m2b) gap len=.4mm area=8u
G4 (m 0) gap len=20mm area=80u

W1 (e1 0 m1 0) winding turns=200
W2 (e2 0 m2b 0) winding turns=200
W3a (e3 0 m3a m3b) winding turns=100
W3b (e4 0 m3b 0) winding turns=10

// Electrical circuit
Vsi (0 vs) vsource type=sine ampl=1 freq=1k
Vi (vr 0) vsource type=pwl wave=[0 0 .3m 15 7m 2]

Vin (vi 0 vs 0 vr 0) pvcvs coeffs=[0 0 0 0 1]
R1 (vi e1) resistor r=2
R2 (e2 0) resistor r=2
R3a (e3 0) resistor r=2
R3b (e4 0) resistor r=2

// Save outputs
save C1:B C1:H C2:B C2:H C3:B C3:H e1 R1:1 R2:1 R3a:1

// Analyses:
sineResp tran stop=6.75ms
```

FIGURE 5 *Graph of input voltage and current waveforms versus time.*



In addition, the implementation of the complex Jiles-Atherton core model is described. Two difficulties of implementing the Jiles-Atherton model is it is implicit and is given in terms of incremental quantities. The model is implemented in Verilog-A[2], which nat-

FIGURE 6 *Graph of magnetization versus magnetic field intensity.*



urally handles implicit equation, and a technique is given that eliminates the incremental nature of the model.

7.1 If You Have Questions

If you have questions about what you have just read, feel free to post them on the *Forum* section of *The Designer's Guide Community* website. Do so by going to *www.designers-guide.org/Forum*.

Acknowledgements

I would like to thank Robert Buckles for pointing out an error in the text.

References

[1] Williams, M. C. *A CAD Model for Nonlinear Magnetics*. Master's Thesis, University of South Florida, April 1994.

[2] Jiles, D. C. and D. L. Atherton. Theory of ferromagnetic hysteresis. *Journal on Magnetism and Magnetic Materials*, 61, (1986): 48.

[3] *Verilog-AMS Language Reference Manual: Analog & Mixed-Signal Extensions to Verilog HDL*, version 2.1. Accellera, January 20, 2003. Available from *www.accellera.com*. An abridged version is available from *www.verilog-ams.com* or *www.designers-guide.org/VerilogAMS*.

[4] Kenneth S. Kundert. *The Designer's Guide to Verilog-AMS*. Kluwer Academic Publishers, 2004.

[5] Langevin, M. *Ann. de Chemistry et Physics*., 5, (1905): 70.

2. Verilog is a registered trademark of Cadence Design Systems licensed to Accellera.

[6] Jiles, D. C. and D. L. Atherton. Ferromagnetic hysteresis. *IEEE Transactions on Magnetics*, March 19 (1983): 2183.

[7] Jiles, D. C. and J. B. Thoelke. Theory of ferromagnetic hysteresis: determination of model parameters from experimental hysteresis loops. *IEEE Transactions on Magnetics*, September 5 (1989): 3928.