

# *Noise in Mixers, Oscillators, Samplers, and Logic*

## An Introduction to Cyclostationary Noise

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The proliferation of wireless and mobile products has dramatically increased the number and variety of low power, high performance electronic systems being designed. Noise is an important limiting factor in these systems. The noise generated is often cyclostationary. This type of noise cannot be predicted using SPICE, nor is it well handled by traditional test equipment such as spectrum analyzers or noise figure meters, but it is available from the new RF simulators.

The origins and characteristics of cyclostationary noise are described in a way that allows designers to understand the impact of cyclostationarity on their circuits. In particular, cyclostationary noise in time-varying systems (mixers), sampling systems (switched filters and sample/holds), thresholding systems (logic circuitry), and autonomous systems (oscillators) is discussed.

**Search Terms**


Oscillator phase noise, jitter, noise folding, modulated noise, time-varying noise, cyclostationary noise.

*This document contains the slides and speaker notes for a presentation on cyclostationary noise given at the Custom Integrated Circuits Conference in May of 2000. The companion paper is also available from [designers-guide.org/theory](http://designers-guide.org/theory).*

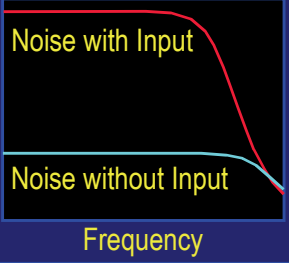
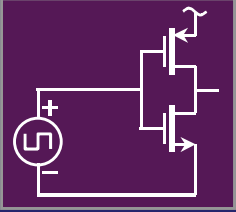
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## 1.0 Motivation



### Motivation



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- What is the effect of a time-varying bias point?
  - Why does the pulse induce more noise?
  - How much information can we get from the spectrum?
  - How to model the noise?

SPICE noise analysis is not able to compute valid noise results for many common classes of circuit for which noise is of interest. Noise in these circuits is of different character than that can be computed by SPICE. We show why and how in this presentation. We explain how this noise differs from the noise discussed in introductory text books on noise, and how this difference affect your circuits.

## 2.0 What is Noise?

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### What is Noise?

$$v_n(t) = v(t) + n(t)$$

- Noise signals are stochastic
  - Small random variation versus time
  - Repeated identical trials give slightly different results
  - A group of trials is an ensemble

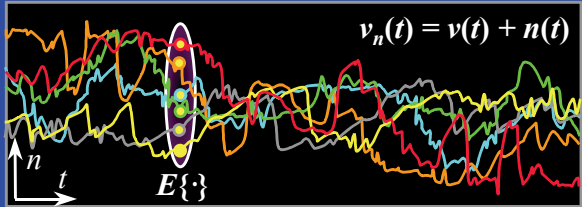
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Noise free systems are deterministic, meaning that repeating the same experiment produces the same result. Noisy systems are stochastic — repeating the same experiment produces slightly different results each time. An experiment is referred to as a trial. A group of experiments is referred to as an ensemble of trials, or simply an ensemble.

### 3.0 Ensemble Averages

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## Ensemble Averages



- Expectation  $E\{\cdot\}$  is average over many trials
- Mean:  $E\{n(t)\} = 0$  and  $E\{v_n(t)\} = v(t)$
- Variance:  $\text{var}\{n(t)\} = E\{n(t)^2\}$  is noise power
- Autocorrelation:  $R_v(t, \tau) = E\{v(t)v(t+\tau)\}$
- Power spectral density:  $S(f) = \left\langle \int R_v(t, \tau) e^{j2\pi f\tau} d\tau \right\rangle$

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Assume that  $v_n(t) = v(t) + n(t)$ , where  $v(t)$  is the deterministic signal,  $n(t)$  is the noise, and  $v_n(t)$  is the combined signal.

Noise is characterized using ensemble averages. An average over many trials is referred to as an expectation, and is denoted  $E\{\cdot\}$ . One ensemble average is the mean. The mean of the combined signal is an estimate of the noise free signal,  $E\{v_n(t)\} = v(t)$ . The mean of the noise alone is generally 0,  $E\{n(t)\} = 0$ . The variance,  $\text{var}(n(t)) = E\{n(t)^2\} - E\{n(t)\}^2$ , is a measure of the power in the noise. The autocorrelation,  $R_n(t, \tau) = E\{n(t)n(t-\tau)\}$ , is a measure of how points on the same signal separated by  $\tau$  seconds are correlated. The autocorrelation is related to the variance by  $\text{var}(n(t)) = R_n(t, 0)$ . The Fourier transform of the autocorrelation function averaged over  $t$  is the time-averaged power spectral density, or PSD. It is the power in the noise as a function of frequency.

## 4.0 Cyclostationary Noise

**Cyclostationary Noise**

- Periodically modulated noise
  - Noise with periodically varying characteristics
  - Results when large periodic signal is applied to a nonlinear circuit
- Has many names
  - Oscillator phase noise
  - Jitter
  - Noise folding
  - AM or PM noise
  - etc.

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Though the term *cyclostationary noise* may be unfamiliar, it is critically important to most designers. They typically know it by other names: oscillator phase noise, jitter, noise folding, AM & PM noise, etc. By looking at noise using the common theoretical foundation of cyclostationary noise allows designers to better understand and predict all of these manifestations of noise.

## 5.0 White Noise

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### White Noise

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- Noise at each time point is independent
  - Noise is uncorrelated in time
  - Spectrum is white
- Examples: thermal noise, shot noise

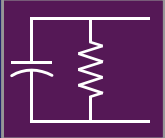
There are two ways of characterizing noise, the power spectral density or PSD,  $S(f)$ , and the autocorrelation function,  $R(t, \tau)$ . These two representations are related by the Fourier transform.

Completely uncorrelated noise is known as white noise. For white noise the PSD is a constant and the autocorrelation function is an impulse function centered at 0.

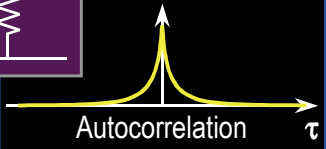
## 6.0 Colored Noise

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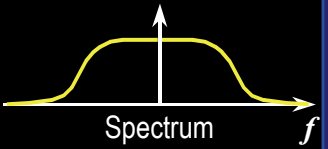
### Colored Noise



$R(\tau) \xleftarrow{\text{Fourier Transform}} S(f) \xrightarrow{\text{Fourier Transform}}$



Autocorrelation  $\tau$



Spectrum  $f$

- Noise is correlated in time because of time constant
- Spectrum is shaped by frequency response of circuit
- Noise at different frequencies is independent (uncorrelated)

**Time correlation  $\leftrightarrow$  Frequency shaping**

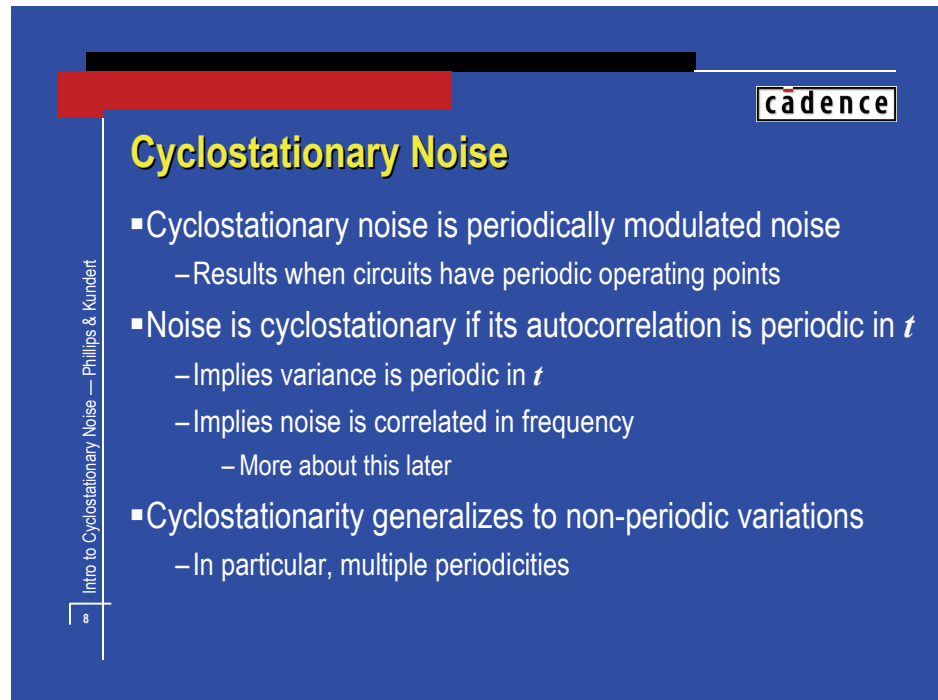
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Energy storage elements cause the circuit to exhibit a frequency response that is shaped; it is not flat with frequency. As such, it causes the PSD to be shaped. This is referred to as coloring the noise. Colored noise has a PSD that varies with frequency.

Energy storage elements also cause the noise to be correlated. This occurs simply because noise produced at one point in time is stored in the energy storage element, and comes out some time later as determined by the time constant of the circuit. This results in the autocorrelation function having nonzero width.

Notice that in this case, both the spectrum is shaped and the noise is correlated over time. This is a general property, shaping the noise in the frequency domain implies that the noise is correlated in time, and visa versa.

## 7.0 Cyclostationary Noise



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### Cyclostationary Noise

- Cyclostationary noise is periodically modulated noise
  - Results when circuits have periodic operating points
- Noise is cyclostationary if its autocorrelation is periodic in  $t$ 
  - Implies variance is periodic in  $t$
  - Implies noise is correlated in frequency
    - More about this later
- Cyclostationarity generalizes to non-periodic variations
  - In particular, multiple periodicities

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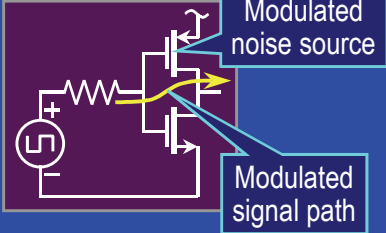
Cyclostationary noise results when noise is periodically modulated. This occurs in any nonlinear circuit that is driven by, or exhibits, large periodic signals.

Noise is cyclostationary if, and only if, its autocorrelation function  $R(t, \tau)$  is periodic in  $t$ . Since the variance is  $R(t, 0)$ , the instantaneous noise power also periodic in  $t$ . Also shown later in this presentation is that noise being cyclostationary also implies that the noise is correlated versus frequency.

Cyclostationarity naturally generalizes to non-periodic variation, in particular, when there are multiple periodicities (also known as quasiperiodicity).



## 8.0 Origins of Cyclostationary Noise



The diagram shows a common-emitter amplifier circuit. It includes a DC bias source (represented by a battery symbol), a resistor, and a transistor. A yellow arrow indicates the signal path from the input to the output. A blue arrow points to a noise source (represented by a lightning bolt symbol) that is modulated by the periodic bias current. The output of the circuit is labeled as 'Modulated signal path'.

**Origins of Cyclostationary Noise**

- Modulated (time-varying) noise sources
  - Periodic bias current generating shot noise
  - Periodic variation in resistance of channel generating thermal noise
- Modulated (time-varying) signal path
  - Modulation of gain by nonlinear devices and periodic operating point


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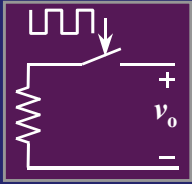
Cyclostationary noise is generated by circuits with periodic operating points. The time-varying operating point modulates the noise generated by bias-dependent noise sources, and modulates the transfer function from the noise source to the output. Both result in cyclostationary noise at the output.

As suggested by the name, modulated noise sources can be modeled by modulating the output of stationary noise sources.


## 9.0 Cyclostationary Noise vs. Time



### Cyclostationary Noise vs. Time



Noisy Resistor  
& Clocked Switch




- Noise transmitted only when switch is closed
- Noise is shaped in time

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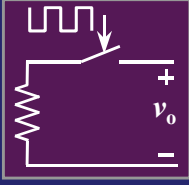
This is a simple example of cyclostationary noise. A periodically operating switch between the noise source (the resistor produces white thermal noise) and the observer causes the output noise to vary periodically.

It can be said that cyclostationary noise is “shaped in time”.

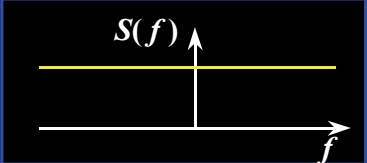
## 10.0 Cyclostationary Noise vs. Frequency



### Cyclostationary Noise vs. Frequency



Noisy Resistor  
& Clocked Switch



- No dynamic elements  $\Rightarrow$  no memory  $\Rightarrow$  no coloring
- Noise is uncorrelated in time
- Spectrum is white
- Cannot see cyclostationarity with time-average spectrum
  - Time-averaged PSD is measured with spectrum analyzer

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With no energy storage elements the noise is completely uncorrelated (noise at a particular time is uncorrelated with the noise at any previous time) and therefore is white, even though it is cyclostationary. One cannot tell that noise is cyclostationary by just observing a simple PSD.

The traditional form of a PSD is referred to as the *time-averaged PSD*. It is what is measured by a spectrum analyzer when the modulation frequency of the cyclostationary noise is well beyond the resolution bandwidth of the analyzer, meaning that it cannot track the periodic variations of the noise. As a result, it presents the time-average spectrum of the noise.

11.0 Cyclostationary Noise vs. Time & Frequency

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### Cyclostationary Noise vs. Time & Frequency

- Sample noise every  $T$  seconds
  - $T$  is the cyclostationarity period
  - Noise versus sampling phase  $\phi$
- Useful for sampling circuits
  - S/H
  - SCF

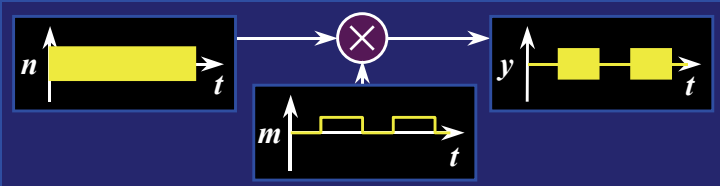
An alternative to the time-average PSD is the instantaneous PSD. It captures the time-varying nature of cyclostationary noise. One conceptually measures a instantaneous PSD by sampling the cyclostationary noise at the noise modulation frequency. The sampling process must be in exact synchronism with the noise modulation process and be offset in phase by  $\phi$ . The result will be a random discrete-time noise process for which we will compute the PSD. As a side note, since it is a discrete-time random process, its spectrum is periodic in  $f_0$ , the sampling frequency. The phase offset  $f$  is then swept over an entire cycle and the process repeated. The result is a family of PSDs for the sampled noise parameterized over  $f$ — the instantaneous PSD.

Because of the sampling nature of the sample & holds and switched-capacitor filters, the instantaneous PSD is often the best way of characterizing noise in these circuits.

## 12.0 Cyclostationary Noise is Modulated Noise

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### Cyclostationary Noise is Modulated Noise



- If noise source is  $n(t)$  and modulation is  $m(t)$ , then
- In time domain, output  $y(t)$  is found with multiplication
 
$$y(t) = m(t) n(t)$$
- In the frequency domain, use convolution
 
$$Y(f) = \sum_k M_k N(f - kf_0)$$

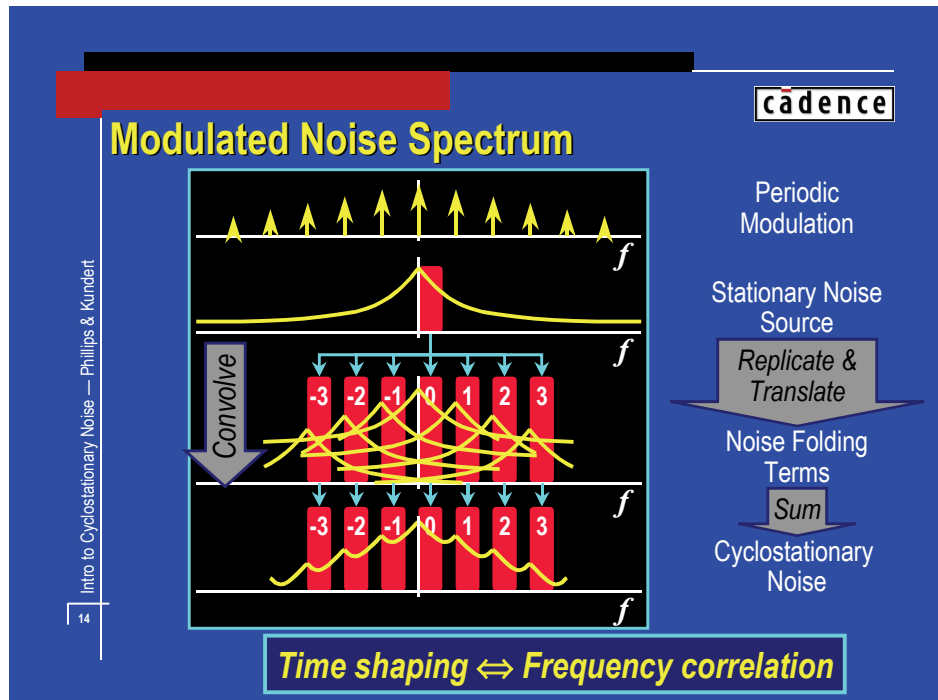
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As mentioned before, cyclostationary noise is another name for modulated noise. Cyclostationary noise can be created by periodically modulating stationary noise. Similarly, cyclostationary noise can always be decomposed into, or modeled as, a stationary noise and a periodic modulation.

In the example shown, the stationary noise is modulation by a periodic pulse train. Recall that in the time domain modulation is performed by simply multiplying the stationary noise by the modulation function. In the frequency domain, the two signals are convolved.

### 13.0 Modulated Noise Spectrum



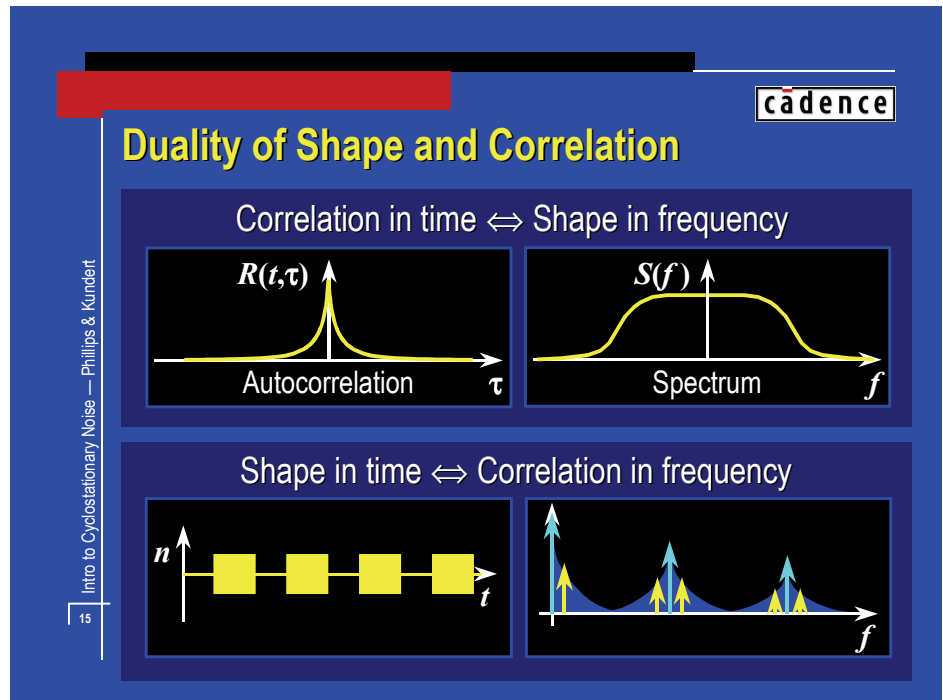
In the example, stationary noise with an arbitrary spectrum is modulated by some periodic signal. This is representative of both ways in which cyclostationary noise is generated (modulated noise sources and modulated signal paths).

Modulation is a convolution in the frequency domain. Thus, the modulation by a periodic signal causes the noise to mix up and down in multiples of the modulation frequency.

Noise from the source at a particular frequency  $f$  is replicated and copies appear at  $f \pm kf_0$  where  $f_0$  is the modulation frequency. Conversely, noise at the output at a particular frequency  $f$  has contributions from noise from the sources at frequencies  $f \pm kf_0$ .

Modulation acts to shape the noise in the time-domain and correlate the noise in the frequency-domain.

## 14.0 Duality of Shape and Correlation



Let's review ...

Recall that

$$\text{shape in frequency} \leftrightarrow \text{correlation in time}$$

Now also see that

$$\text{shape in time} \leftrightarrow \text{correlation in frequency}$$

This is the duality of shape and correlation. These relationships occur because in both cases the quantities on the left and right are related by the Fourier transform.

15.0 Correlations in Cyclostationary Noise

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### Correlations in Cyclostationary Noise

- Noise is replicated and offset by  $kf_0$ 
  - Noise separated by multiples of  $f_0$  is correlated

- With real signals, spectrum is symmetric
  - Upper and lower sidebands are correlated

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As just shown, modulation causes noise to be replicated and translated in frequency. Thus, noise separated by  $kf_0$  is correlated where  $f_0$  is the modulation frequency. Remember, noise folds across DC in a conjugate symmetric manner, so noise in upper and lower sidebands are correlated. In other words, in the top diagram noise is shown at both negative and positive frequencies. This implies a complex phasor representation is being used. When this complex signal is converted to a real signal, the complex conjugate of signals at negative frequencies get mapped to positive frequencies. In this way, the signal at frequencies  $\Delta\omega$  above and below a harmonic are correlated. These frequencies are referred to as upper and lower sidebands of the harmonic.



## 16.0 Sidebands and Modulation

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## Sidebands and Modulation

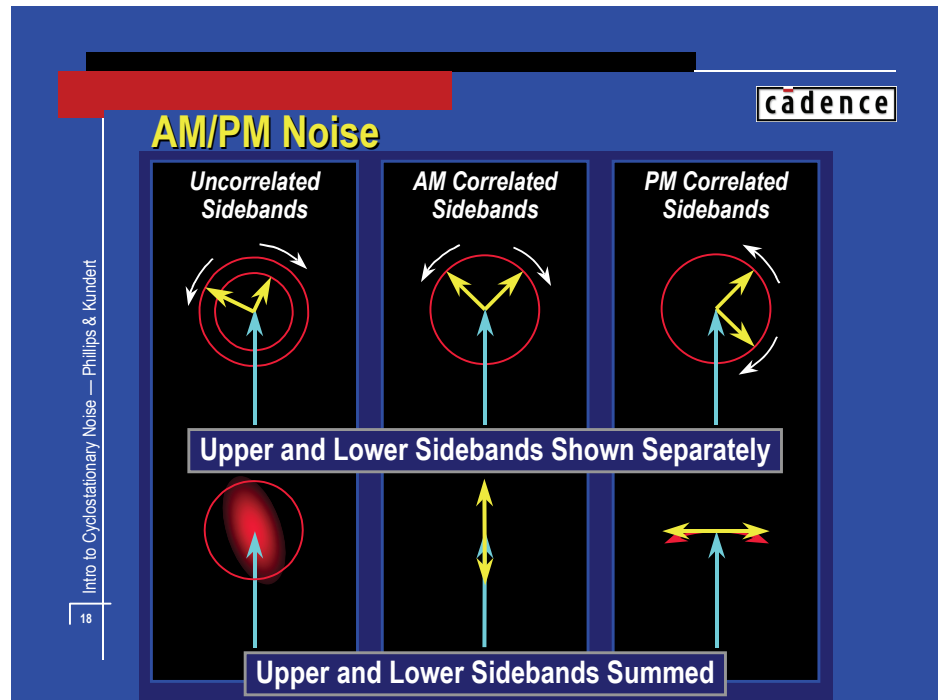
- Correlations in sidebands  $\Leftrightarrow$  AM/PM modulation
- To separate noise into AM/PM components
  - Consider noise sidebands separated from carrier by  $\Delta f$
  - Add sideband phasors to tip of carrier phasor
  - Relative to carrier, one rotates at  $\Delta f$ , the other at  $-\Delta f$

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One can separate noise near the carrier into AM and PM components. Consider the noise sidebands at frequencies  $\Delta f$  from the carrier. Treat both these sidebands and the carrier as phasors. Individually add the sideband phasors to the carrier phasor. The sideband phasors are at a different frequency from the carrier, and so will rotate relative to it. One sideband will rotate at  $\Delta f$ , and the other at  $-\Delta f$ . If the noise is not cyclostationary, then the two sidebands will be uncorrelated. Meaning that their amplitude and phase will vary randomly relative to each other. Combined, the two sideband phasors will trace out an ellipse whose size, shape, and orientation will shift randomly. However, if the noise is cyclostationary, then the sidebands are correlated. This reduces the shifting in the shape and orientation of the ellipse traced out by the phasors. If the noise is perfectly correlated, then the shape and orientation remain unchanged, though its size still shifts randomly.

One can then decompose the ellipse into orthogonal components. The component parallel to the carrier phasor represents the amplitude modulation (AM noise) and the component perpendicular to the carrier represents the phase modulation (PM noise).

## 17.0 AM/PM Noise



With cyclostationary noise, the noise in various sidebands is correlated. Depending on the magnitude and phase of the correlation, the noise at the output of the circuit can be AM noise, PM noise, or some combination of the two. For example, oscillators almost exclusively generate PM noise near the carrier whereas noise on the control input to a variable gain amplifier results almost completely in AM noise at the output of the amplifier.

When considering the noise about a carrier frequency, the noise can be decomposed into AM and PM components. Having one component of noise dominate over the other is a characteristic of cyclostationary noise. Stationary noise can also be decomposed into AM and PM components, but there will always be equal amounts of both.

There are three cases illustrated. The first represents the case where stationary noise is added to the carrier. In this case the sidebands will have the same variance (average length) but are completely uncorrelated. The length and phase randomly vary, and in the short term trace out an ellipse that wobbles and changes its shape. But over the long term the noise envelope is a circle, which implies that there is an equal amount of AM and PM in the noise. This is always true with stationary noise.

When the noise is cyclostationary the sidebands will be correlated, which results in the AM component dominating over the PM, or vice versa.

## 18.0 Noise + Compression = Phase Noise

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### Noise + Compression = Phase Noise

- Stationary noise contains equal AM & PM components
- With compression or saturation
  - Carrier causes gain to be periodically modulated
  - Modulation acts to suppress AM component of noise
    - Leaving PM component
- Examples
  - Oscillator phase noise
  - Jitter in logic circuits
  - Noise at output of limiters

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It is a general rule that when stationary noise is passed through a stage undergoing compression or saturation, the noise at the output is predominantly phase noise. Stationary noise contains equal amounts of amplitude and phase noise. Passing it through a stage undergoing compression cause the AM noise to be suppressed, leaving mainly the PM noise.

## 19.0 Ways of Characterizing Cyclostationary Noise

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### Ways of Characterizing Cyclostationary Noise

- Time-average power spectral density
  - Hides cyclostationarity
  - Useful when cyclostationary nature of noise is not important
- Time-domain descriptions (noise vs. phase)
  - Completely characterizes cyclostationary noise
  - Thresholds and jitter, sampled data systems
- Spectrum with correlations
  - Noise and correlations versus frequency
  - Completely characterizes cyclostationary noise
  - Decomposition into AM/PM components

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More review ...

There are three common methods of characterizing cyclostationary noise.

The time-average power spectral density is similar to what would be measured with a conventional spectrum analyzer. Since it has a very small effective input bandwidth, it ignores correlations in the noise and so ignores the cyclostationary nature of the noise (assuming that the frequency of the variations in the noise is much higher than the bandwidth of the analyzer). This is the primary output from SpectreRF's PNoise analysis.

The second method is to use the spectrum along with information about the correlations in the noise between sidebands. This is a complete description of the cyclostationarity in the noise. It is used when considering the impact of cyclostationary noise from one stage on a subsequent synchronous stage. This would be the case if two stages were driven by the same LO or clock, or if the output of one stage caused the subsequent stage to behave nonlinearly. From this form it is relatively easy to determine the amount of power in the AM or PM components of the noise. SpectreRF outputs the correlations between sidebands if *noisetype=correlations*.

The third method is to track the noise at a point in phase, or noise versus phase. The noise at a point in phase is defined as the noise in the sequence of values obtained if a periodic signal is repeatedly sampled at the same point in phase during each period. It is useful in determining the noise that will result when converting a continuous-time signal to a discrete-time signal. It is also useful when determining the jitter associated with a noisy signal crossing a threshold. SpectreRF outputs the noise versus phase if *noisetype=timedomain*.

## 20.0 When Can Cyclostationarity be Ignored?

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### When Can Cyclostationarity be Ignored?

Stage 1

→

Stage 2

- Stage 1 produces cyclostationary noise, but we only know the time-average spectrum.
  - Can we use it to predict the noise of the system?
- If we know the noise figure of both stages, can we compute the noise figure of the system?

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Sometimes it is only possible to determine the time-average PSD. So an important question is, when is it good enough? In other words, if you have a stage that is producing cyclostationary noise that drives another stage, and you know the time-average spectrum of the noise produced by the first stage, when is that enough to allow you to predict the effect of that noise on the subsequent stage.

It is enough to know the time-average spectrum of the noise produced by the first stage if the subsequent stage effectively averages the noise at its input. There are two cases where that occurs.

1. When the subsequent stage is a narrowband filter.
2. When the subsequent stage is non-synchronous with the one producing the cyclostationary noise.

Consider the case where one stage drives another. The first stage generates cyclostationary noise, but we only know the time-average PSD. In what situations do we know enough to characterize the noise of the whole system and when do we not. More specifically, assume that we know the noise figures of both stages, can we combine the noise figures using the standard formula to compute the noise figure of the entire system? These formulas assume that the noise produced by each stage is stationary, which is not true in this case.

## 21.0 Removing Cyclostationarity

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### Removing Cyclostationarity

- Filtering can remove cyclostationarity
  - Keeps noise-folding terms, but removes correlated frequencies
  - Filtering must be single-sided with  $BW < f_0/2$

- Examples: mixer w/ filter, SCF w/ anti-aliasing filter
- Can use time-averaged power spectral density
- Can use noise figure

If a stage that generates cyclostationary noise is followed by a filter whose passband is constrained to a single sideband (the passband does not contain a harmonic and has a bandwidth of less than  $f_0/2$ , where  $f_0$  is the fundamental frequency of the cyclostationarity), then the output of the filter will be stationary. This is true because noise at any frequency  $f_1$  is uncorrelated with noise at any other frequency  $f_2$  as long as both  $f_1$  and  $f_2$  are within the passband.

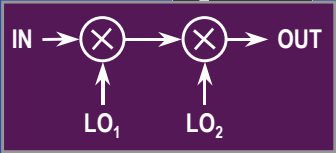
In other words, the filter eliminates noise at any frequency that might be possibly be correlated with noise within the passband of the filter and so the resulting noise is not cyclostationary.

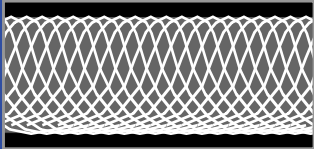
## 22.0 Ignoring Cyclostationarity

### Ignoring Cyclostationarity

*Filtering in Disguise*

- Subsequent stage is non-synchronous
  - Different reference oscillator (as with spectrum analyzer)
- Subsequent stage is synchronous, but over many periods
  - Differing frequencies  $f_1$  and  $f_2$  with  $f_1 / f_2 = n/m$  and  $n, m$  large (mixer chain)





Rolling phase from period-to-period averages noise

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Consider a stage that generates cyclostationary noise with modulation frequency  $f_1$  that is followed by a stage whose transfer characteristics vary periodically at a frequency of  $f_2$  (such as a mixer, sampler, etc.). Assume that  $f_1$  and  $f_2$  are non commensurate (there is no  $f_0$  such that  $f_1 = n f_0$  and  $f_2 = m f_0$  with  $n$  and  $m$  both integers). Then there is no way to shift  $f_1$  by a multiple of  $f_2$  and have it fall on correlated copy of itself. As a result, the cyclostationary nature of the noise at the output of the first stage can be ignored (with regard to its effect on the subsequent stage, the noise from the first stage can be treated as being stationary and we can characterize it using the time-average power spectral density).

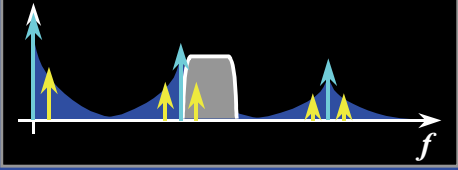
If  $f_1$  and  $f_2$  are commensurate, but  $m$  and  $n$  are both large and have no common factors, then many periods of  $f_1$  and  $f_2$  are averaged before the exact phasing between the two repeats. Again, the cyclostationary nature of the noise at the output of the first stage can be ignored.

## 23.0 When to Use the Time-Averaged PSD

cadence

### When to Use the Time-Averaged PSD

- When subsequent stage is non-synchronous
  - Spectrum analyzer
- Subsequent stage runs at a sufficiently different frequency  $f_1$ 
  - $f_0/f_1 = N/M$ , both  $M, N$  large ( $> 4$ ) with no common factors
- When filtering eliminates correlation in the noise
  - SSB filter with  $BW < f_0/2$



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You are free to use the time-averaged power spectral density (PSD) when the cyclostationary nature of the noise will be eliminated or ignored by subsequent stages. Filtering eliminates the cyclostationary nature of noise, converting it to stationary noise, if the filter is a single-sideband filter with bandwidth less than  $f_0/2$ . The cyclostationary nature of the noise is ignored if the subsequent stage is not synchronous with the noise, or if it is synchronous but running at a sufficiently different frequency so that averaging serves to eliminate the cyclostationarity.



## 24.0 When Knowing Time-Average PSD of a Stage is not Enough

**cadence**

### When Knowing Time-Average PSD of a Stage is not Enough

- When the subsequent stage varies synchronously with the first
  - When subsequent stage shares the same LO or clock
    - Switched-capacitor filter followed by S&H and/or ADC
  - When output signal causes subsequent stage to respond nonlinearly
    - Oscillator driving mixer
    - Chain of logic gates
    - Large interferer in receiver chain
- In these cases, must use complete representation
  - Noise versus time, or spectrum with correlations

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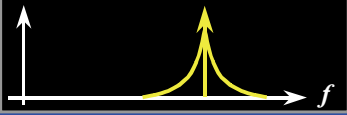
When a stage producing cyclostationary noise drives a subsequent stage that has a time-varying transfer function that is synchronous with the first, then ignoring the cyclostationary nature of the noise from the first (using the time-average PSD) generates incorrect results. One common situation where this occurs is when a switched-capacitor filter is followed by a sample-and-hold, and both are clocked at the same rate (or a multiple of the same rate). Another common situation is when the first stage produces a periodic signal that is large enough to drive the subsequent stage to behave nonlinearly. In this case, large periodic output signal that modulates the gain of the subsequent stage in synchronism with the cyclostationary noise produced by the first stage. This occurs with an oscillator drives the LO port of a mixer, when one logic gate drives another, or when a large interfering signal drives both stages into compression.

In these situations, one must consider the cyclostationary nature of the noise produced in the first stage when determining the overall noise performance of the stages together. Usually this must be done by either simulating or measuring the two stages together rather than apart.

## 25.0 Oscillator Phase Noise

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### Oscillator Phase Noise



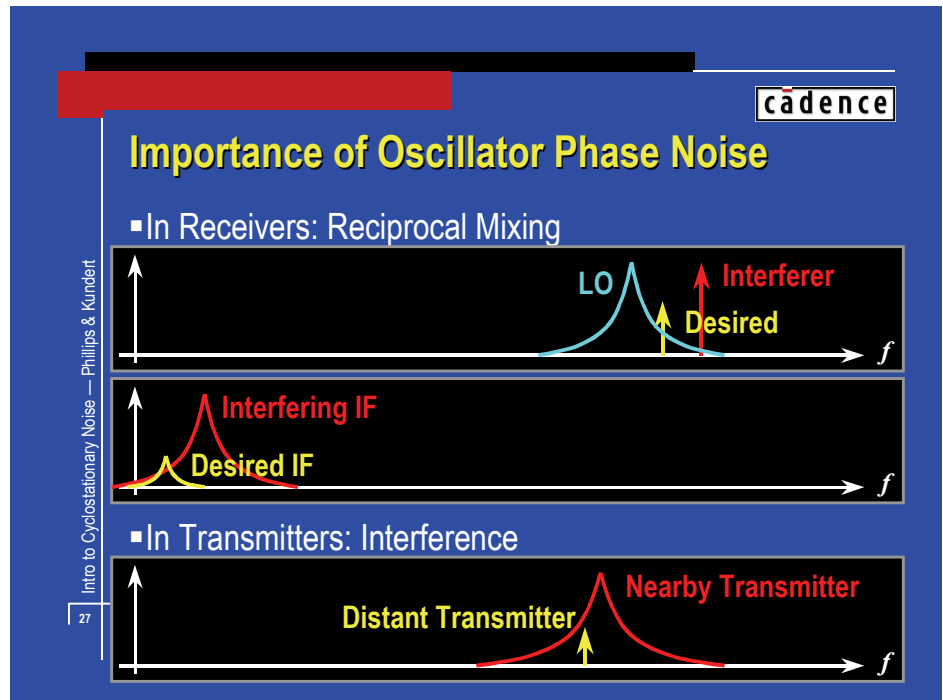
- High levels of noise near the carrier
  - Exhibited by all autonomous systems
  - Noise is predominantly in phase of oscillator
    - Cannot be eliminated by passing signal through a limiter
  - Noise is very close to carrier
    - Cannot be eliminated by filtering
- Oscillators have stable limit cycles
  - Amplitude is stabilized; amplitude variations are suppressed
  - Phase is free to drift; phase variations accumulate

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Another important class of cyclostationary noise is oscillator phase noise.

It is the nature of all autonomous systems, such as oscillators that they produce relatively high levels of noise at frequencies close to the oscillation frequency. Because the noise is close to the oscillation frequency, it cannot be removed with filtering without also removing the oscillation signal. It is the nature of nonlinear oscillators that this noise be predominantly in the phase of the oscillation. Thus, the noise cannot be removed by passing the signal through a limiter.

## 26.0 Importance of Oscillator Phase Noise




In a receiver, the phase noise of the LO can mix with a large interfering signal from a neighboring channel and swamp out the signal from the desired channel even though most of the power in the interfering IF is removed by the IF filter. This process is referred to as *reciprocal mixing*.

Similarly, phase noise in the signal produced by a nearby transmitter can interfere with the reception of a desired signal at a different frequency produced by a distant transmitter.

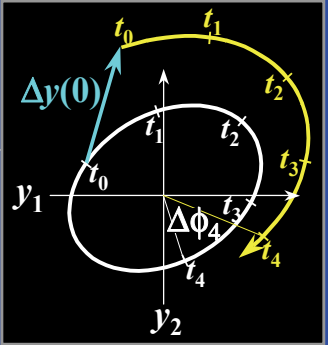
27.0 The Oscillator Limit Cycle

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## The Oscillator Limit Cycle

- Solution trajectory follows a stable orbit,  $y$ 
  - Amplitude is stabilized, but phase is free to drift
- If perturbed with an impulse
  - Response is  $\Delta y$
  - Decompose into amplitude and phase
    - $\Delta y(t) = (1 + \alpha(t))y(t + \phi(t)/2\pi f_c)$
    - Amplitude deviation,  $\alpha(t)$ , is resisted by mechanism that controls output level
      - $\alpha(t) \rightarrow 0$  as  $t \rightarrow \infty$
    - Phase deviation,  $\phi(t)$ , accumulates
      - $\phi(t) \rightarrow \Delta\phi$  as  $t \rightarrow \infty$



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Consider plotting the capacitor voltage against the inductor current in a resonant oscillator. In steady state, the trajectory is a stable limit cycle. Now consider perturbing the oscillator with an impulse  $x = \delta(t)$  and assume that the response to the perturbation is  $\Delta y$ . Separate  $\Delta y$  into amplitude and phase variations,

$$\Delta y(t) = (1 + \alpha(t))y(t + \phi(t)/2\pi f_c) - y(t).$$

where  $\Delta y(t)$  represents the noisy output voltage of the oscillator,  $\alpha(t)$  represents the variation in amplitude,  $\phi(t)$  is the variation in phase, and  $f_c$  is the oscillation frequency.

Since the oscillator is stable and the duration of the disturbance is finite, the deviation in amplitude eventually decays away and the oscillator returns to its stable orbit ( $\alpha(t) \rightarrow 0$  as  $t \rightarrow \infty$ ). In effect, there is a restoring force that tends to act against amplitude noise. This restoring force is a natural consequence of the nonlinear nature of the oscillator and at least partially suppresses amplitude variations.

Since the oscillator is autonomous, any time-shifted version of the solution is also a solution. Once the phase has shifted due to a perturbation, the oscillator continues on as if never disturbed except for the shift in the phase of the oscillation. There is no restoring force on the phase and so phase deviations accumulate. A single perturbation causes the phase to permanently shift ( $\phi(t) \rightarrow \Delta\phi$  as  $t \rightarrow \infty$ ).

## 28.0 The Oscillator Limit Cycle (cont.)

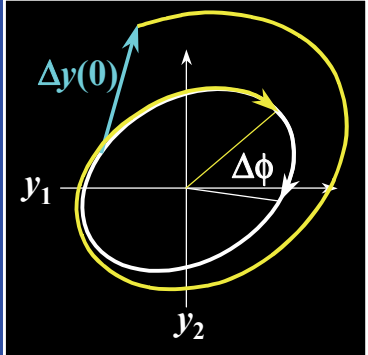
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### The Oscillator Limit Cycle (cont.)

- If perturbed with an impulse
  - Amplitude deviation dissipates  
 $\alpha(t) \rightarrow 0$  as  $t \rightarrow \infty$
  - Phase deviation persists  
 $\phi(t) \rightarrow \Delta\phi$  as  $t \rightarrow \infty$
  - Impulse response for phase is approximated with a step  $s(t)$
- For arbitrary perturbation  $u(t)$ 

$$\phi(t) \propto \int s(t-\tau)u(\tau) d\tau = \int u(t) dt$$

$$S_\phi(f) = \frac{S_u(f)}{(2\pi f)^2}$$



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On the previous slide it was pointed out that  $\alpha(t) \rightarrow 0$  as  $t \rightarrow \infty$  and  $\phi(t) \rightarrow \Delta\phi$  as  $t \rightarrow \infty$ . If we neglect any short term time constants, it can be inferred that the impulse response of the phase deviation  $\phi(t)$  can be approximated with a unit step  $s(t)$ . The phase shift over time for an arbitrary input disturbance  $u$  is

$$\phi(t) \sim \int s(t-\tau)u(\tau)d\tau = \int u(t) dt,$$

or the power spectral density (PSD) of the phase is

$$S_\phi(f) \sim S_u(f)/(2\pi f)^2$$

This represents another way of explaining why oscillator noise is primarily phase noise and why the noise grows at frequencies close to the carrier frequency.

### 29.0 1/Δf<sup>2</sup> Amplification of Noise in Oscillator

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## 1/Δf<sup>2</sup> Amplification of Noise in Oscillator

- Noise from any source
  - Is amplified by 1/Δf<sup>2</sup> in power
  - Is amplified by 1/Δf in voltage
  - Is converted to phase
  
- Phase noise in oscillators
  - Flicker phase noise ~ 1/Δf<sup>3</sup>
  - White phase noise ~ 1/Δf<sup>2</sup>

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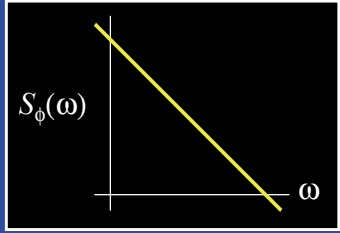
As shown previously, the gain from a noise source to the output phase is 1/Δf<sup>2</sup> in power or 1/Δf in voltage. As such, flicker noise, whose power spectral density is proportional to 1/Δf in power, ends up producing a 1/Δf<sup>3</sup> region in power in the power spectral density of the phase at the output of the oscillator. Similarly, white noise ends up producing a 1/Δf<sup>2</sup> region in power in the power spectral density of the phase at the output of the oscillator.

### 30.0 Difference Between $S_\phi$ and $S_v$ Noise

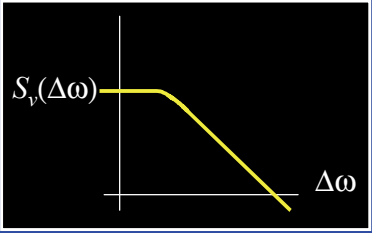
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## Difference Between $S_\phi$ and $S_v$ Noise

- Oscillator phase drifts without bound
  - $S_\phi(\omega) \rightarrow \infty$  as  $\omega \rightarrow 0$
- Voltage is bounded, must remain on limit cycle
  - Total signal power is independent of noise level
  - Corner frequency is proportional to noise level
  - PNoise computes  $S_v(\Delta\omega)$  but does not predict corner



$S_\phi(\omega)$



$S_v(\Delta\omega)$

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There are two different ways of characterizing noise in the same oscillator.  $S_\phi$  is the spectral density of the phase and  $S_v$  is the spectral density of the voltage.  $S_v$  contains both amplitude and phase noise components, but with oscillators the phase noise dominates except at frequencies far from the carrier and its harmonics.  $S_v$  is directly observable on a spectrum analyzer, whereas  $S_\phi$  is only observable if the signal is first passed through a phase detector. Another measure of oscillator noise is  $L$ , which is simply  $S_v$  normalized to the power in the fundamental.

As  $t \rightarrow \infty$  the phase of the oscillator drifts without bound, and so  $S_\phi(\omega) \rightarrow \infty$  as  $\omega \rightarrow 0$ . However, even as the phase drifts without bound, the excursion in the voltage is limited by the diameter of the limit cycle of the oscillator. Therefore, as  $\Delta\omega \rightarrow 0$  the PSD of  $v$  flattens out. The more phase noise, broader the linewidth (the higher the corner frequency), and the lower signal amplitude within the linewidth. This happens because the phase noise does not affect the total power in the signal, it only affects its distribution. Without phase noise,  $S_v(\omega)$  is a series of impulse functions at the harmonics of the oscillation frequency. With phase noise, the impulse functions spread, becoming fatter and shorter but retaining the same total power.

The voltage noise  $S_v$  is considered small outside the linewidth and thus can be accurately predicted using small signal analyses. Conversely, the voltage noise within the linewidth is large and cannot be predicted with small signal analyses. Thus, small signal noise analysis, such as is available from RF simulators, is valid only up to the corner frequency (it does not model the corner itself).

## 31.0 Oscillator Phase Noise

**cadence**

### Oscillator Phase Noise

- Comparing phase over long periods
  - Phase drifts randomly over long periods
  - Drift randomizes phase, signal appears stationary
  - Smeared correlation in frequency
  - Occurs in radar with long time-of-flight
- Comparing phase over short periods
  - Phase is not randomized, signal appears cyclostationary
  - Occurs in
    - RF circuits
    - Radar with short time-of-flight

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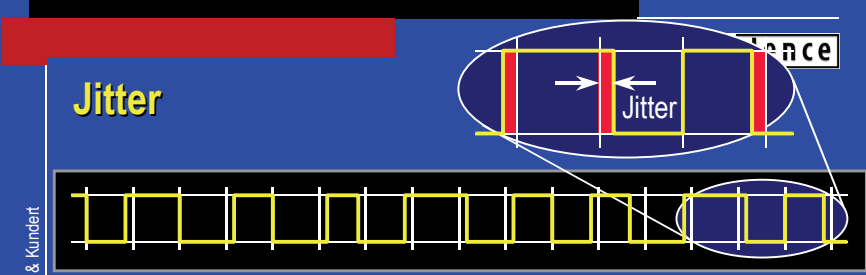
The phase of an oscillator is a random walk process. Thus, if you know the phase of an oscillator at some point, that will give you information about the phase of the oscillator in the near future but not the distant future. After the phase has been measured, each cycle brings a bit of uncertainty in the phase. That uncertainty accumulated over 1000's of cycles until eventually it builds up to be more than one complete cycle. At this point the phase is completely randomized and you no longer have any information about the phase.

Consider an oscillator used in a radar that is measuring the distance to the moon. Assume that the oscillator generates the signal that is transmitted to the moon, and that during the trip the phase of the oscillator becomes completely randomized. Once the signal bounces off the moon and returns, it is mixed with the oscillator signal again to convert it to baseband. Since the phase has been completely randomized, we can model this system with two different free running oscillators operating at the same frequency, one that generates the transmission signal, and the other that is used for the LO. Thus, the mixer is operating completely asynchronously with the transmit oscillator and so the noise produced by the transmit oscillator looks stationary to the mixer. Thus, over the long term oscillators produce stationary noise.

However, over the short term they produce cyclostationary noise. In a typical RF circuit, such as a LO driving a mixer, the mixer is operating in complete synchronism with the oscillator and so the noise produced by the oscillator appears cyclostationary to the mixer.



## 32.0 Jitter



**Jitter**

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- Jitter is an undesired fluctuation in the timing of events
  - Modeled as a “noise in time”

$$v_j(t) = v(t + j(t))$$

- The time-domain equivalent of phase noise

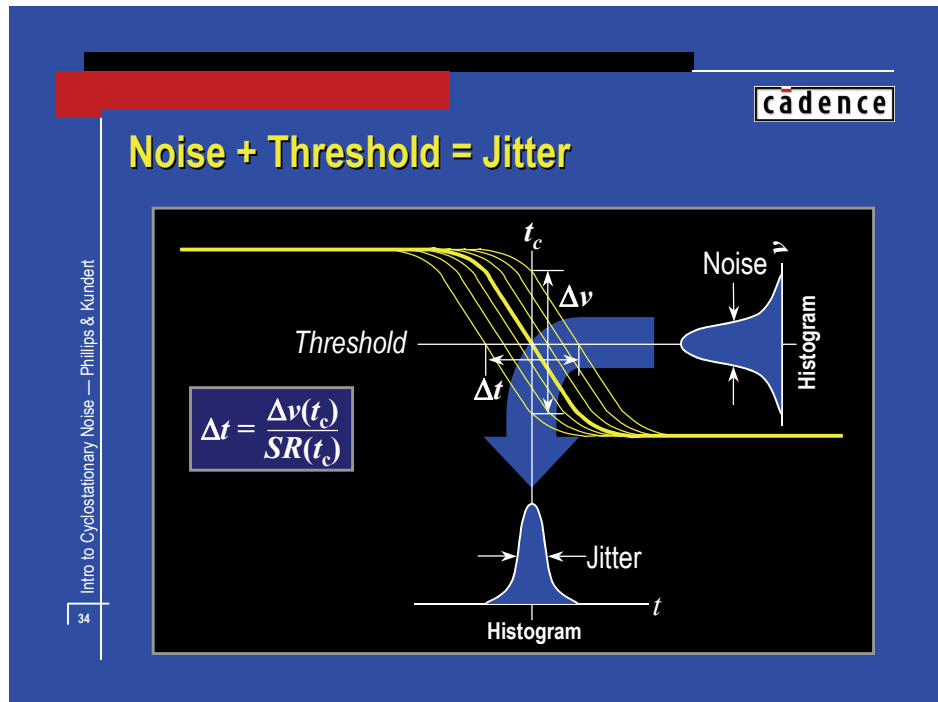
$$j(t) = \phi(t)T / 2\pi$$

- Jitter is caused by phase noise or noise with a threshold

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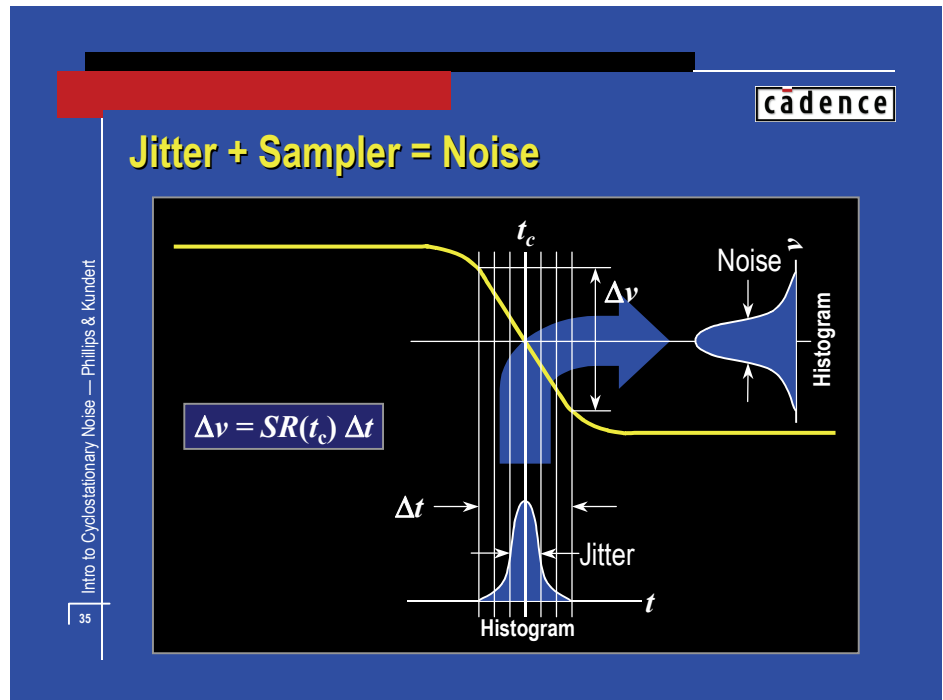
Jitter is an undesired fluctuation in the timing of events. It can be modeled as a noise in time. It is equivalent to phase noise and is used in situations where it is more natural to think of the noise being in time rather than in phase or in signal level. Like phase noise, it serves to degrade system performance.

33.0 Noise + Threshold = Jitter



Noise added to a signal transitioning through a threshold adds jitter to the time at which the signal crosses the threshold. This is the way jitter is created in nonlinear circuits such as digital circuitry. The statistics of the jitter can be computed from the statistics of the noise at the time of the threshold crossing and the slewrate (or time derivative) of the large signal at the threshold crossing.

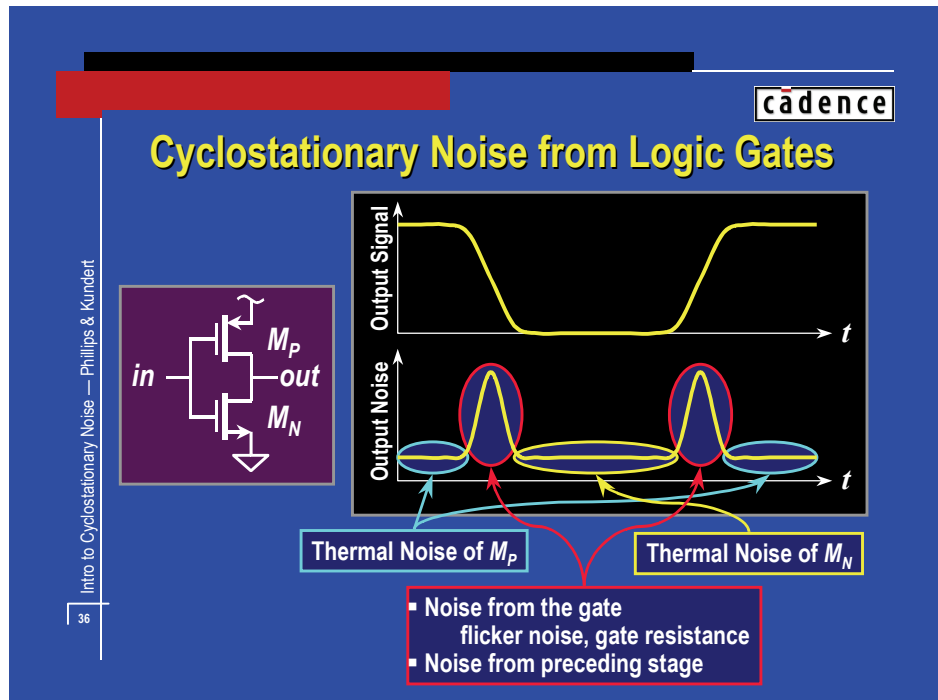
## 34.0 Jitter + Sampler = Noise



Jitter in the time at which a signal is sampled creates noise in the sampled result if the signal is changing at the time it is sampled. This is one way in which noise is generated when converting continuous-time signals are converted to discrete-time signals. The statistics of the noise can be computed from the statistics of the jitter at the time of the sampling and the slewrate (or time derivative) of the input signal at the time of the sampling.

If one samples a constant valued signal, jitter in the time at which the sampling occurs does not create noise in the output. Thus, during flat portions of waveforms, an uncertainty in the sampling time creates no noise

### 35.0 Cyclostationary Noise from Logic Gates

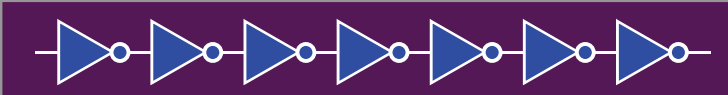


The noise produced by a logic gate comes from different places depending on the state of the output. When the output is high and  $M_P$  is on, the output is insensitive to small changes on the input and the noise at the output is predominantly due to the thermal noise from the channel of  $M_P$ . When the output is low, the situation is reversed and most of the output noise is due to the thermal noise from the channel of  $M_N$ . When the output is transitioning, thermal noise from both  $M_P$  and  $M_N$  contributes to the output. In addition, the output is sensitive to small changes in the input, in fact, noise from the input is amplified before reaching the output. Thus, noise from the input tends to dominate over the thermal noise from the channels of  $M_P$  and  $M_N$  in this region. Noise at the input includes noise from the previous stage and thermal noise from the gate resistance. In addition, with significant current flowing in the transistors, flicker noise from the channel also contributes.

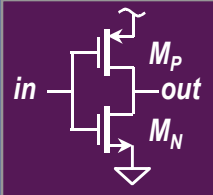
## 36.0 Noise in a Chain of Logic Gates

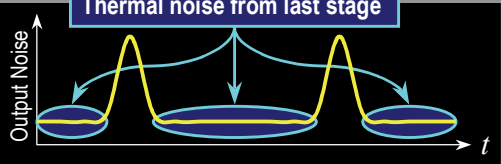
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### Noise in a Chain of Logic Gates



- Thermal noise of last stage often dominates the time-average noise spectrum — but not the jitter!
  - Is ignored by subsequent stages
  - Must be removed when characterizing jitter





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The total output noise power of a gate is dominated by the thermal noise produced by the output devices if the gate spends most of its time with an unchanging output. This noise is usually ignored by subsequent stages. Thus, using the time-averaged spectral density to characterize the noise in a gate is misleading. Only the noise produced by a gate at the point where its output crosses the threshold of the subsequent stage should be taken into account when characterizing the jitter of the gate.

## 37.0 Characterizing Jitter in Logic Gate

**cadence**

### Characterizing Jitter in Logic Gate

- If noise vs. time can be determined
  - Find noise at peak
  - Integrate over all frequencies
  - Divide total noise by slewrate at peak
- If noise contributors can be determined
  - Measure noise contributions from stage of interest on output of subsequent stage
  - Integrate over all frequencies
  - Divide total noise by slewrate at peak
  - Alternatively, find phase noise contributions, convert to jitter
- Otherwise
  - Build noise-free model of subsequent stage
  - Apply noise-contributors approach

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There are several different methods that can be used to determine the jitter of a logic block using a simulator such as SpectreRF. They all assume that the circuit is driven, so as to produce a periodic response, and that a cyclostationary noise analysis is performed about that periodic operating point.

The first approach can be used if the simulator is capable of determining the instantaneous noise power at a particular point in time (in this case the time of the threshold crossing) (SpectreRF can do this). Simply determine the time where the output signal crosses the threshold. Then find both the instantaneous noise power and the slewrate of the output signal at that time. Then convert the noise power to a noise voltage and divide through by the slewrate to determine the jitter.

The second approach can be used if the contributions of individual noise sources can be identified. In this case, the circuit is simulated along with the subsequent stage, or the stage it is expected to drive. The subsequent stage must be a thresholding circuit, and so the noise at the output will only contain contributions from the stage of interest if they contribute as jitter. Any noise from the stage of interest that does not manifest as jitter at the output of the subsequent stage will be blocked by the thresholding nature of the subsequent stage. To determine the jitter at the output due to the stage of interest, divide the average total noise at the output contributed by the stage of interest and divide by the RMS value of the derivative of the output signal.

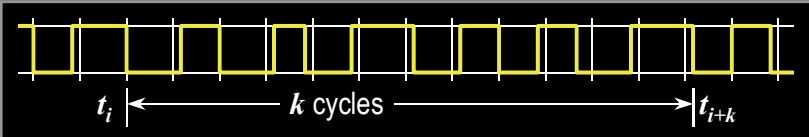
Finally, the third approach is similar to the second, except that rather than following the stage of interest with the actual subsequent stage, you replace it with a model that produces no noise of its own. One can create such a model in the form of a limiter using Verilog-A (for proper operation it must have finite gain in its active region). In this way,

only the stage of interest is producing noise and so one can use the total output noise rather than the individual noise contributions. Again, one divides the average total output noise by the RMS value of the time-derivative of the output signal to compute the jitter.

## 38.0 Characterizing Jitter

cadence

### Characterizing Jitter



- $J_k$  —  $k$ -cycle jitter
  - The deviation in the length of  $k$  cycles

$$J_k(i) = \sqrt{\text{var}(t_{i+k} - t_i)}$$

- For driven circuits jitter is input- or self-referenced
  - $t_i$  is from input signal,  $t_{i+1}$  is from output signal, or
  - $t_i$  and  $t_{i+1}$  are both from output signal
- For autonomous circuits jitter is self-referenced
  - $t_i$  and  $t_{i+1}$  both from output signal

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A common way to characterize jitter is  $k$ -cycle jitter,  $J_k$ . It represents the variation in the length of  $k$  cycles. If there is no jitter, then  $J_k$  will be 0, if the jitter on each cycle is completely uncorrelated with the jitter on previous cycles, then  $J_k$  will be constant with  $k$ , and if the jitter is correlated from cycle-to-cycle, then  $J_k$  will vary as a function of  $k$ .

For driven circuits, there are two different ways to measure  $k$ -cycle jitter.  $J_k$  is input-referred if a transition on the input signal starts the measurement interval.  $J_k$  is self-referred if a transition on the output signals starts the measurement interval. The results achieved using these two different approaches varies slightly because the output-referred jitter includes an extra noisy transition when compared to the input-referred jitter. Input-referred jitter is generally used for driven circuits.

With autonomous circuits there is no input transition that can be used to start the measurement interval, so self-referred jitter is used on oscillators.

39.0 Jitter in Simple Driven Circuits (Logic)

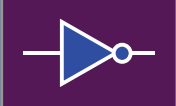


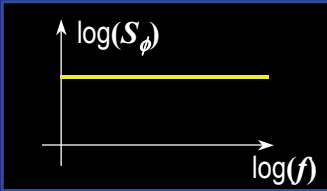
## Jitter in Simple Driven Circuits (Logic)

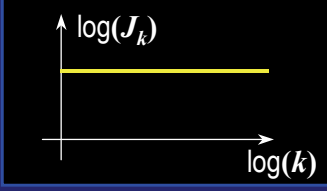
- Assumptions
  - Memory of circuit is shorter than cycle period
  - Noise is white (NBW  $\gg 1/T$ )
  - Input-referenced measurement
- Implications
  - Each transition is independent
  - No accumulation of jitter
  - $J_k = \Delta t$  for all  $k$

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The next few slides attempt to give an feeling for what to expect with  $J_k$  on several important classes of circuits, First, consider a simple driven circuit such as an inverter. Assume that the bandwidth of the circuit is well beyond the input frequency, that the noise sources in the circuit are all white, and that  $J_k$  is measured using a input-referred measurement. In this case, the jitter on each transition is independent and uncorrelated to that on any other transition and  $J_k = \Delta t$  for all  $k$ .




40.0 Jitter in Autonomous Circuits (Ring Osc, ...)

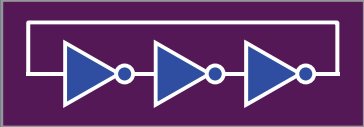
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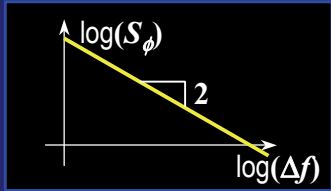
## Jitter in Autonomous Circuits (Ring Osc, ...)

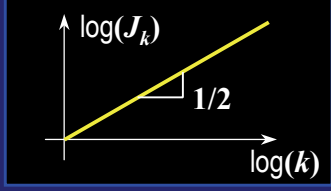
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- Assumptions
  - Memory of circuit is shorter than cycle period
  - Noise is white (NBW >> 1/T)
  - Self-referenced measurement
- Implications
  - Each transition relative to previous
  - Jitter accumulates
  - $J_k = \sqrt{k} \Delta t$









Next, consider a simple autonomous circuit such as an ring oscillator. Assume that the noise sources in the circuit are all white and that  $J_k$  is measured using a self-referred measurement. In this case, the jitter variance accumulates from cycle-to-cycle, and so  $J_k = k^{1/2} \Delta t$ .

41.0 Jitter in PLLs

Intro to Cyclostationary Noise — Phillips & Kundert

## Jitter in PLLs

- Assumptions
  - Memory of circuit is longer than cycle period
  - Noise is white (or NBW  $\gg 1/T$ )
  - Self-referenced measurement
- Implications
  - Jitter accumulates for  $k$  small
    - $J_k = \sqrt{k} \Delta t$
  - No accumulation for  $k$  large
    - $J_k = \Delta T$  where
    - $\Delta T = \frac{\Delta t}{\sqrt{2\pi f_L}}$

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Finally, consider a circuit that combines the characteristic of the previous two circuits such as a phase locked loop. At low frequencies, the gain of the loop acts to suppress the jitter produced by the voltage-controlled oscillator (VCO). In this case, if we assume the reference is free of jitter, then the jitter is due to the phase-frequency detector & charge pump (PFD/CP) and the frequency divider (FD), which are wideband driven circuits. Thus, at low frequencies the noise is approximately flat with frequency. For the same reasons,  $J_k$  is independent of  $k$  for large  $k$ . At high frequencies, the low-pass filter (LPF) essentially opens the loop. Thus the noise and jitter characteristics of the output are basically those of the VCO alone. Thus, for large  $\Delta f$  the noise is proportional to  $J_k f^{-2}$  and for small  $k$   $J_k$  is proportional  $k^{1/2}$ .

## 42.0 Summary

cadence

### Summary

- Cyclostationary noise is modulated noise
  - Found where ever large periodic signals are present
  - Mixers, oscillators, sample-holds, SCF, logic, etc.
- Cyclostationary noise is correlated versus frequency
  - Leads to AM and PM components in noise
- Several ways of characterizing cyclostationary noise
  - Time-average spectrum
    - Incomplete, hides cyclostationarity
  - Noise versus time and frequency
    - Useful for sample-holds, SCF, logic, etc.
  - Noise versus frequency with correlations (AM & PM noise)
    - Useful for oscillators, mixers, etc.

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### 42.1 If You Have Questions

If you have questions about what you have just read, feel free to post them on the *Forum* section of *The Designer's Guide Community* website. Do so by going to [designers-guide.org/forum](http://designers-guide.org/forum).